

displayed in fig. 5 shows that our value for  $\beta$  agrees within the experimental error with theirs. Our measurements give thus a full experimental justification of the model of Rice *et al.* in its form represented by eq. (5).

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## A RIGOROUS APPROACH TO A MARKOFFIAN MASTER EQUATION

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## Synopsis

The part  $\Phi_0(t)$  of the statistical operator (or density function) which is relevant for the description of macroscopic dynamics is treated. The few mathematical properties of the solution of Zwanzig's generalized master equation which are important for the deduction of a markoffian master equation for  $\Phi_0(t)$  are pointed out. On the basis of such results the conditions under which markoffian macroscopic dynamics exists are discussed. A comparison with the results of I. Prigogine's school is finally made.

1. *Introduction.* In a previous paper<sup>1)</sup> we discussed the problem of the approach to equilibrium for the macro-observables of an insulated macroscopic system.

The limit for  $t \rightarrow +\infty$  of the solution of a generalized master equation (G.M.E.) was proved to be given by the microcanonical ensemble; for this result only very general mathematical properties of the Laplace-transformed nucleus of the G.M.E. are relevant. One expects that this approach to equilibrium is well described, at a macroscopic level, by a markoffian master equation; this is a much more difficult problem to study, since it requires a more detailed analysis of the dynamics of macro-observables.

In this paper we elaborate on some peculiar mathematical features of the G.M.E. which make it possible for a macroscopic markoffian evolution to exist. The main tool for this analysis is a systematic use of the spectral representation of the relevant operators in a finite-dimensional Liouville-space.

The mathematical results, obtained in sections 2 and 3, are discussed in a less rigorous way in section 4 and some physical consequences are obtained. The main conclusion is that a markoffian behaviour exists if a suitably defined characteristic time  $\tau_0$  is microscopic with respect to the time scale